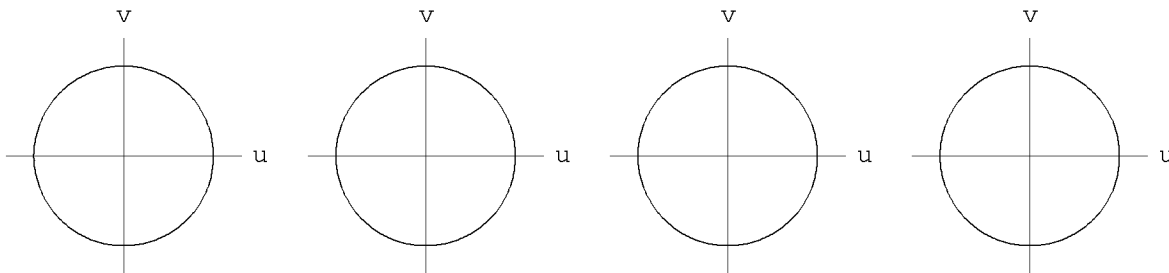


Name _____ raw scaled percent

Math 11 Trimester 2 Practice Exam 1P
Trigonometry

- **Partial credit may be given for correct work. Therefore, it is to your advantage to write clear solutions. If I cannot understand a solution within 90 seconds, then it will receive no partial credit.**
- **Answers must be completely simplified. No denominators may include radicals. All fractions reduced. Arithmetic must be completely performed; e.g. write 9 instead of $\sqrt{81}$.**
- **All angles you write for answers must be written with respect to the angle zero and measured in the positive direction (counter clockwise). For example, write $\theta = \frac{3\pi}{2}$ rather than $\theta = -\frac{\pi}{2}$.**
- **No calculators. All answers must be exact.**

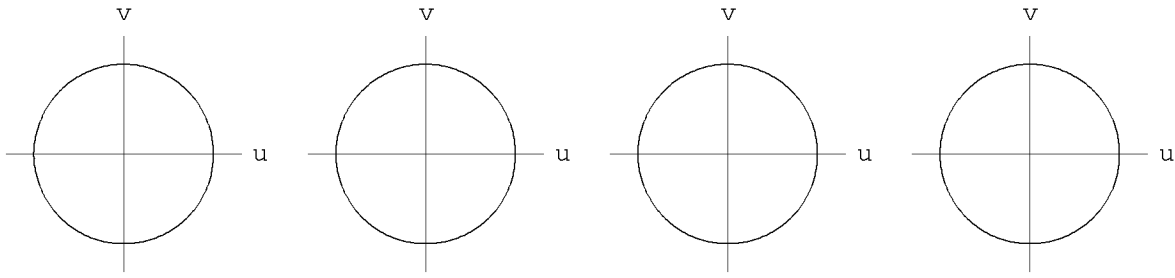


- **A. Find the values of the following.**

[NB: for some of these you will use the identities of side 1 or the unit circle, for others you will use the identities of side 2.]

[1] $\cos \frac{-4\pi}{3}$

[2] $\sin \frac{5\pi}{6}$



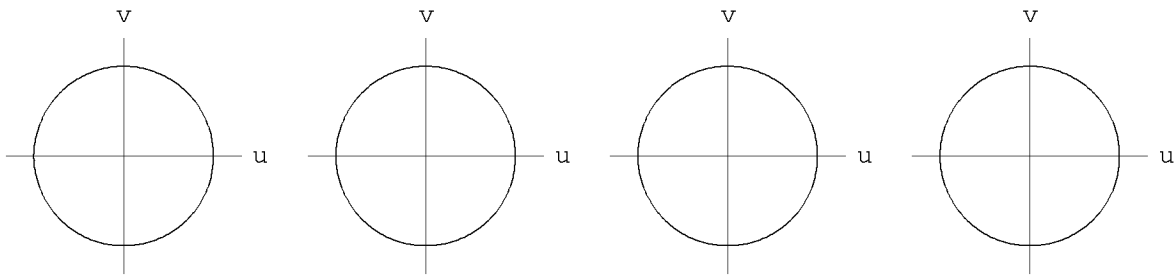
[3] $\cos \frac{\pi}{12}$

[4] $\sin \frac{2\pi}{3}$

[5] $\cos 29\pi$

[6] $\tan \frac{7\pi}{12}$

[7] $\sin 15^\circ$



■ B. Answer the following. Use correct notation.

[1] Find all values of x for which $\sin x = \frac{-\sqrt{3}}{2}$.

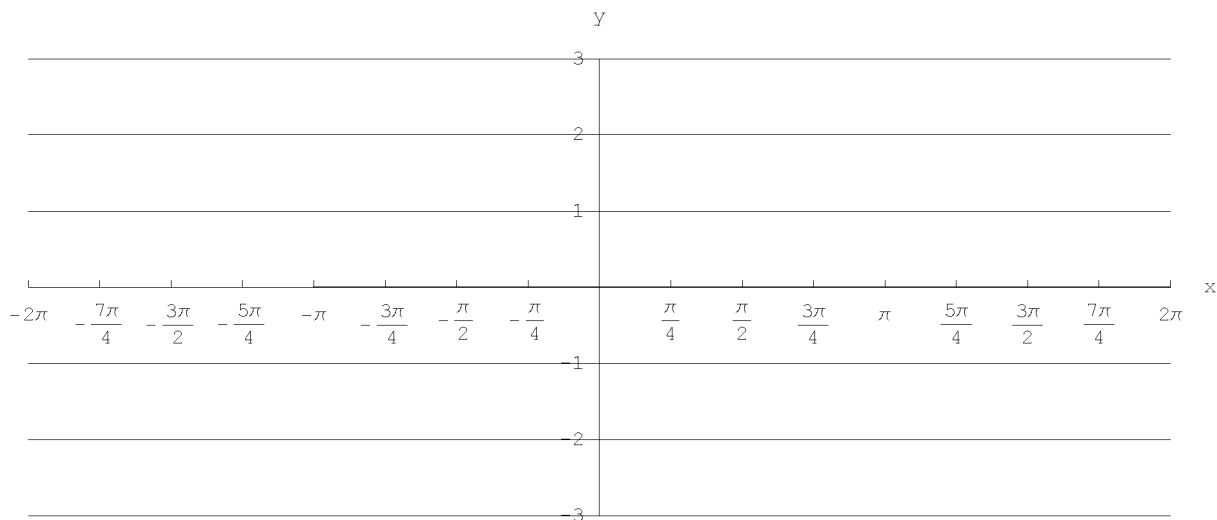
[2] Find all values of x , $0 \leq x < 2\pi$, for which $\sin x \leq \frac{-\sqrt{3}}{2}$.

[3] Find all values of x , $0 \leq x < 2\pi$, for which $\cos x \geq \frac{-\sqrt{3}}{2}$.

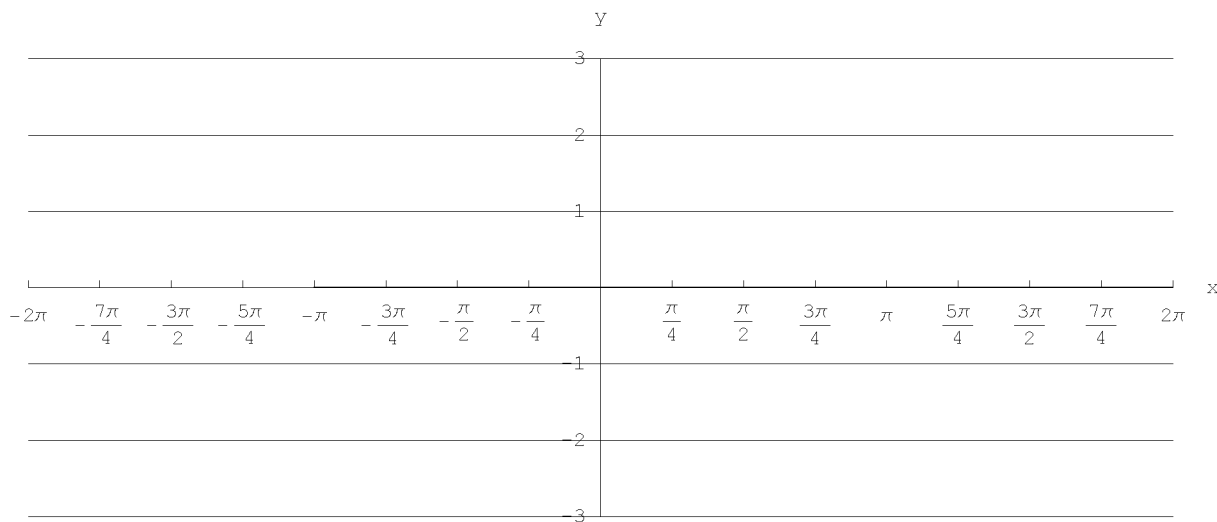
[4] Find all values of x , $0 \leq x < 2\pi$, for which $\tan x > 0$.

■ C. Graph the following (neatly).

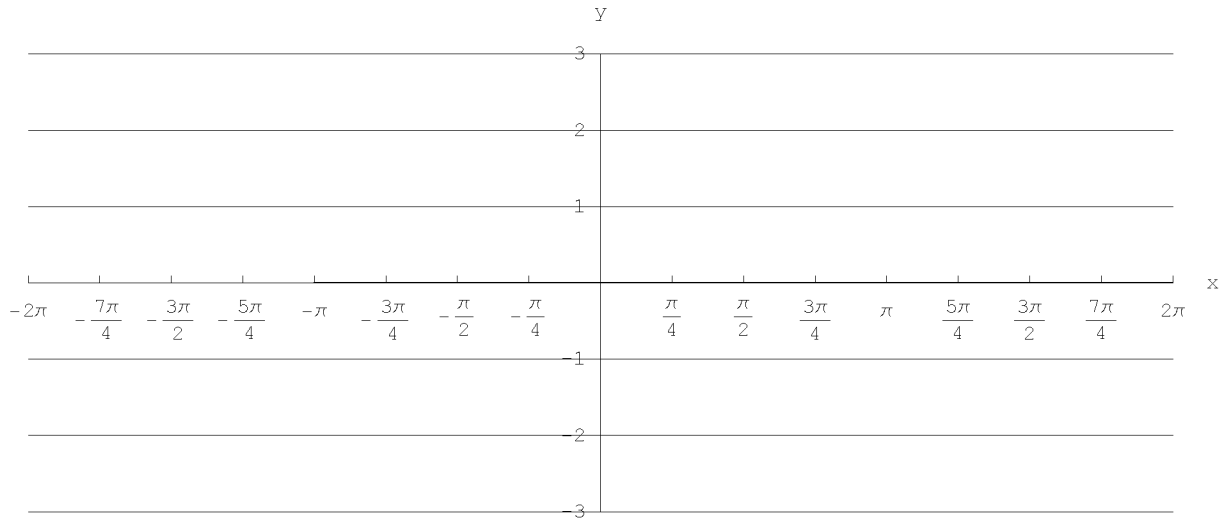
[1] The tangent function



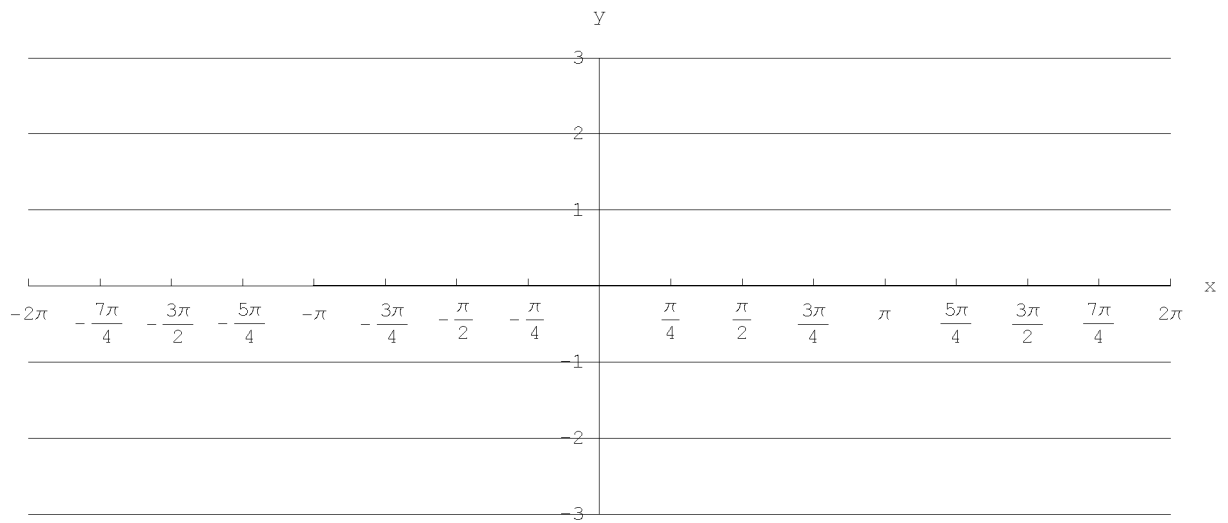
[2] $\sin(x - \frac{\pi}{4})$



[3] $3 \sin 2x$

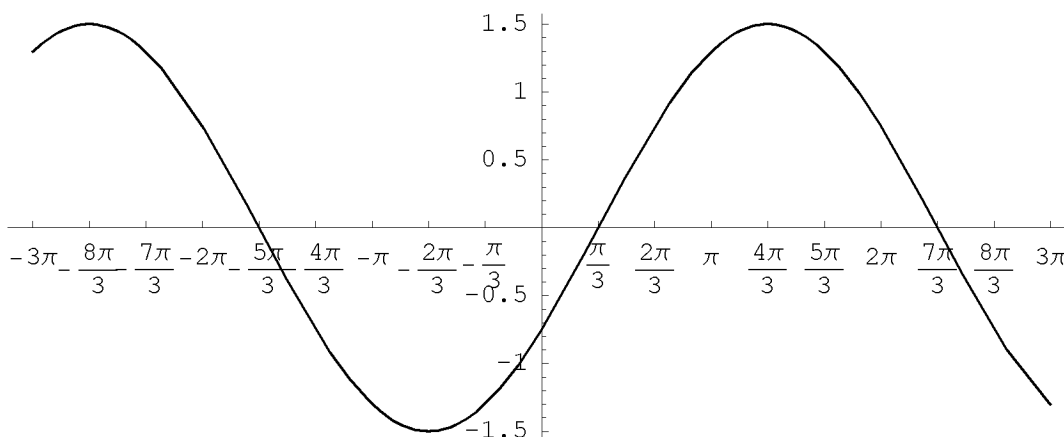


[4] $2 \sin (2x - \frac{\pi}{2})$



■ D. Write the function for each graph.

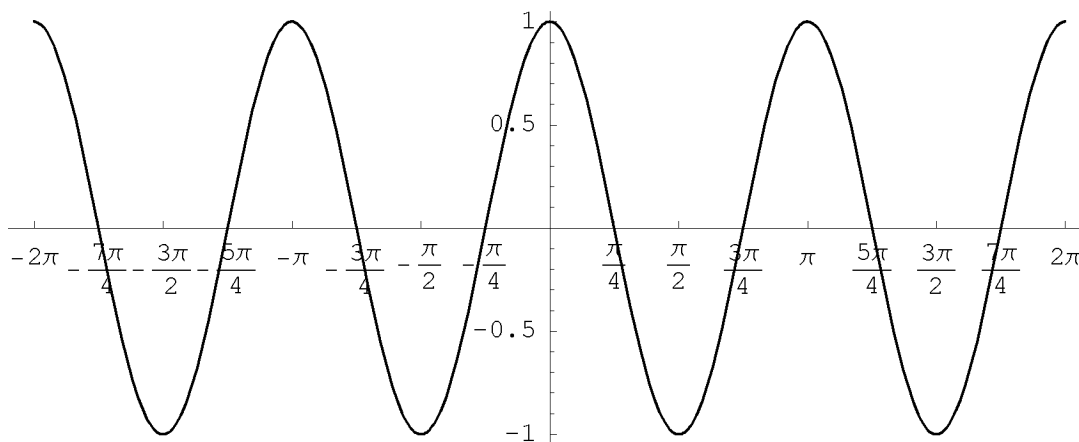
[1] The graph crosses the x-axis at $-\frac{5\pi}{3}, \frac{\pi}{3}, \frac{7\pi}{3}$. It has a maximum of 1.5 and a minimum of -1.5 .



The function of graph [1] is:

Answer

[2] The graph crosses the x-axis at $-\frac{5\pi}{3}, \frac{3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$. It has a maximum of 1.0 and a minimum of -1.0 .



The function of graph [2] is:

Answer

■ **D. Proofs**

[1] Prove any one of the addition theorems that involves only sines and cosines.

[2] Prove: $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

[3] Prove: $\sin \alpha \cos \beta = \frac{1}{2} [\sin (\alpha + \beta) + \sin (\alpha - \beta)]$

[4] Prove: $\cos 2 x = 2 \cos^2 x - 1$

■ E. Answer the following

[1] Find the maximum and minimum values of $y = \sin \theta + \sqrt{3} \cos \theta$ and state the values of θ , $0 \leq \theta < 2\pi$, at which they occur.

[2] Solve for θ , $0 \leq \theta \leq 2\pi$, if $\sin \theta = 2 \sin^2 \theta$.

[3] Solve for x , if $5 \cos 3x - 1 = 4 \cos 3x + 1$

[4] Solve for x , if $\cos 2x - 1 = 3 \cos 2x - 1$

[5] Solve for θ , if $\sin 4\theta = -\sin 3\theta$

■ F. Answer the following. These involve the Law of Sines and the Law of Cosines. Quantities given are exact. Use a calculator to answer correct to 4 significant figures.

[1] Solve $\triangle ABC$ if $\alpha = 47^\circ$, $a = 25$ ft, $b = 30$ ft

[1] Solve $\triangle ABC$ if $a = 56$ ft, $b = 83$ ft, $\gamma = 97^\circ$

Answers Math 11 Trimester 2 Practice Exam 1P

Trigonometry

■ A.

$$[1] \cos\left[-\frac{4\pi}{3}\right] = -\frac{1}{2}$$

$$[2] \sin\left[\frac{5\pi}{6}\right] = \frac{1}{2}$$

$$[3] \cos\left[\frac{\pi}{12}\right] = \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$[4] \sin\left[\frac{2\pi}{3}\right] = \frac{\sqrt{3}}{2}$$

$$[5] \cos[29\pi] = -1$$

$$[6] \tan\left[\frac{7\pi}{12}\right] = -2 - \sqrt{3}$$

$$[7] \sin[15^\circ] = \frac{-1+\sqrt{3}}{2\sqrt{2}}$$

■ B. Answer the following. Use correct notation.

$$[1] \text{ Find all values of } x \text{ for which } \sin x = \frac{-\sqrt{3}}{2}.$$

$$x \in \left\{x \ni x = \frac{4\pi}{3} + 2n\pi, n \in \mathbb{Z}\right\} \cup \left\{x \ni x = \frac{5\pi}{3} + 2n\pi, n \in \mathbb{Z}\right\}$$

$$[2] \text{ Find all values of } x, 0 \leq x < 2\pi, \text{ for which } \sin x \leq \frac{-\sqrt{3}}{2}.$$

$$\frac{4\pi}{3} \leq x \leq \frac{5\pi}{3}$$

$$\left\{x \ni \frac{4\pi}{3} \leq x \leq \frac{5\pi}{3}\right\}$$

$$\left[\frac{4\pi}{3}, \frac{5\pi}{3}\right]$$

$$[3] \text{ Find all values of } x, 0 \leq x < 2\pi, \text{ for which } \cos x \geq \frac{-\sqrt{3}}{2}.$$

$$\left\{x \ni 0 \leq x \leq \frac{5\pi}{6}\right\} \cup \left\{x \ni \frac{7\pi}{6} \leq x \leq 2\pi\right\}$$

$$\left[0, \frac{5\pi}{6}\right] \cup \left[\frac{7\pi}{6}, 2\pi\right]$$

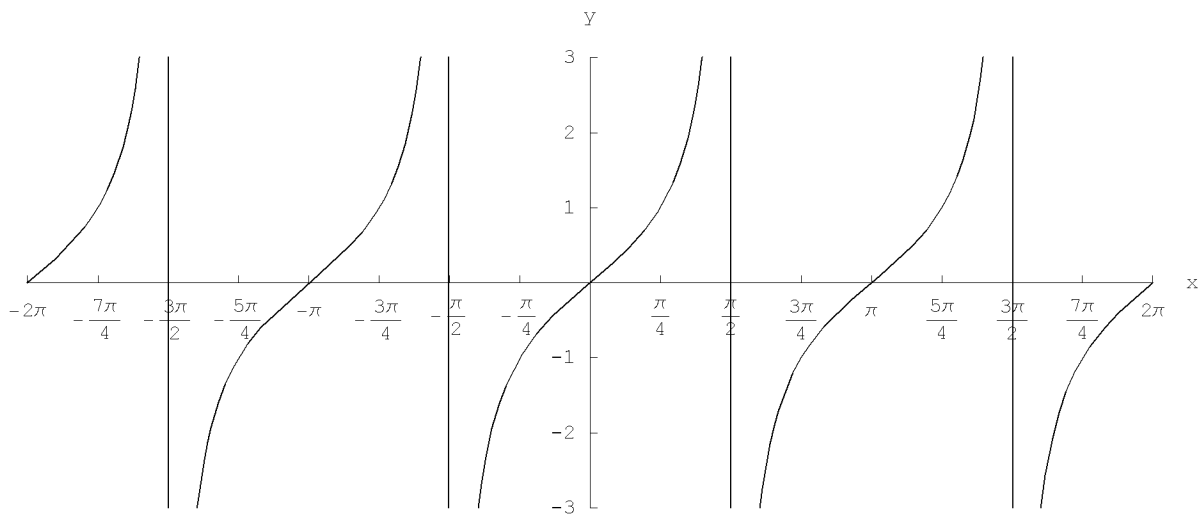
[4] Find all values of x , $0 \leq x < 2\pi$, for which $\tan x > 0$.

$$\{x \ni 0 \leq x \leq \frac{\pi}{2}\} \cup \{x \ni \pi \leq x \leq \frac{3\pi}{2}\}$$

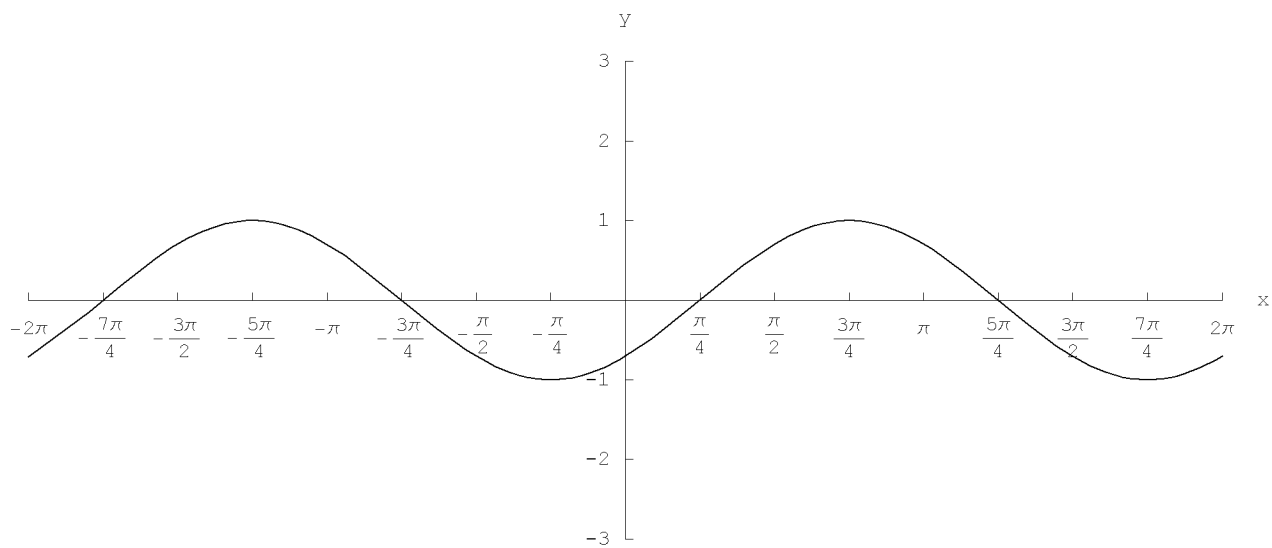
$$[0, \frac{\pi}{2}] \cup [\pi, \frac{3\pi}{2}]$$

■ C. Graph the following (neatly).

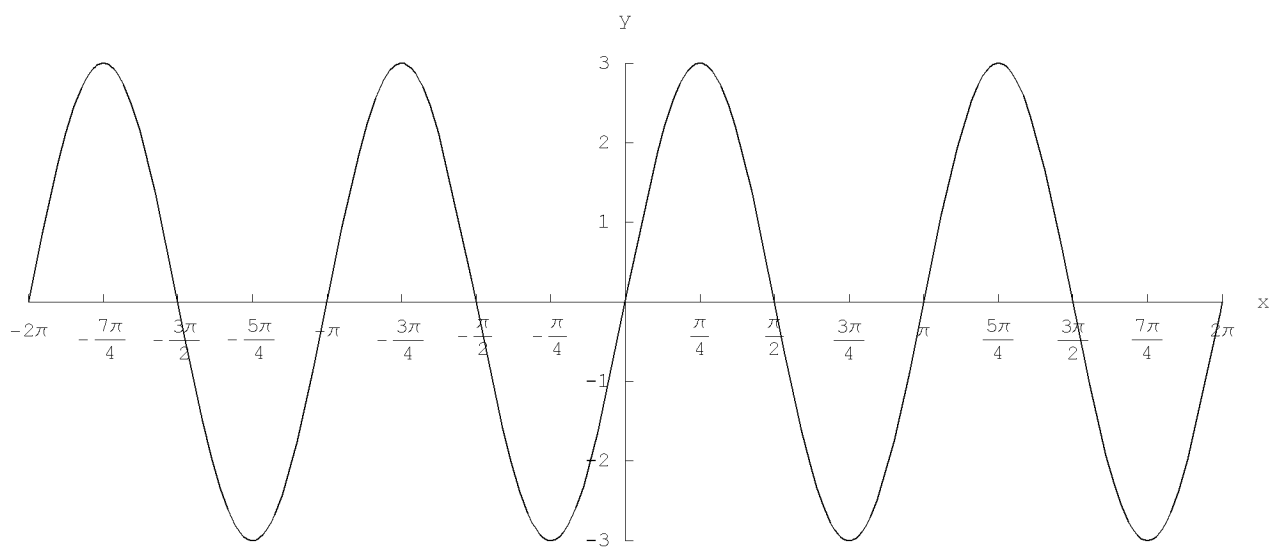
[1] The tangent function



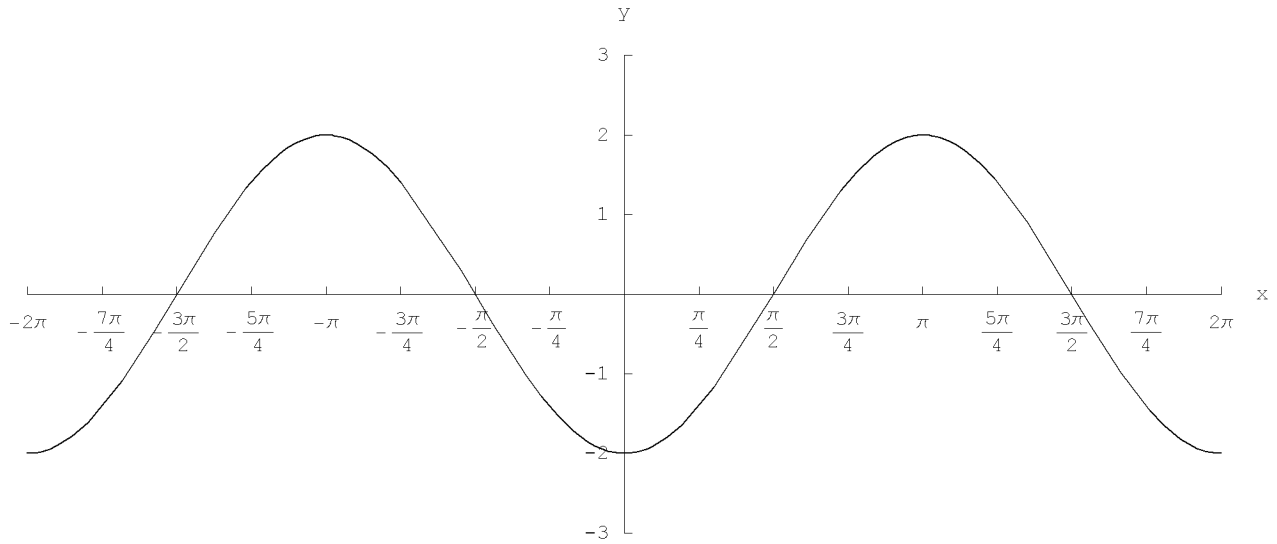
[2] $\sin(x - \frac{\pi}{4})$



[3] $3 \sin 2x$

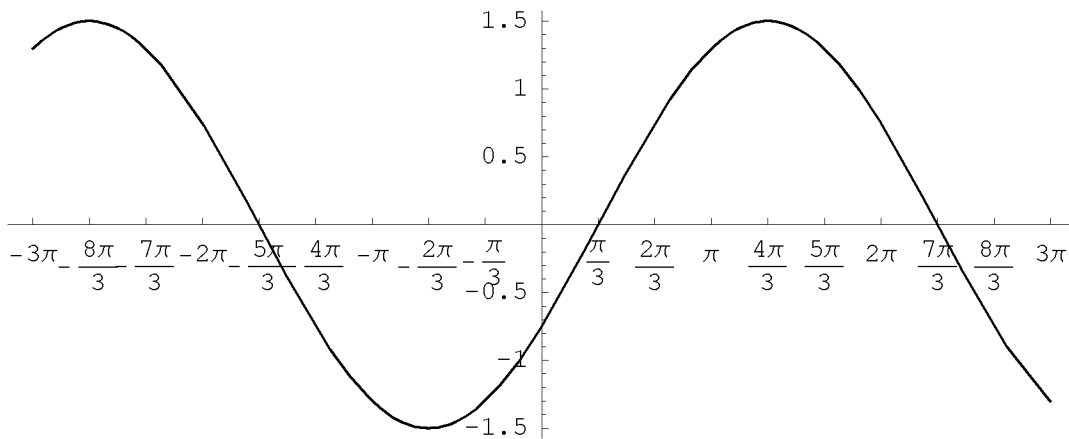


[4] $2 \sin \left(2x - \frac{\pi}{2} \right)$



■ D. Write the function for each graph.

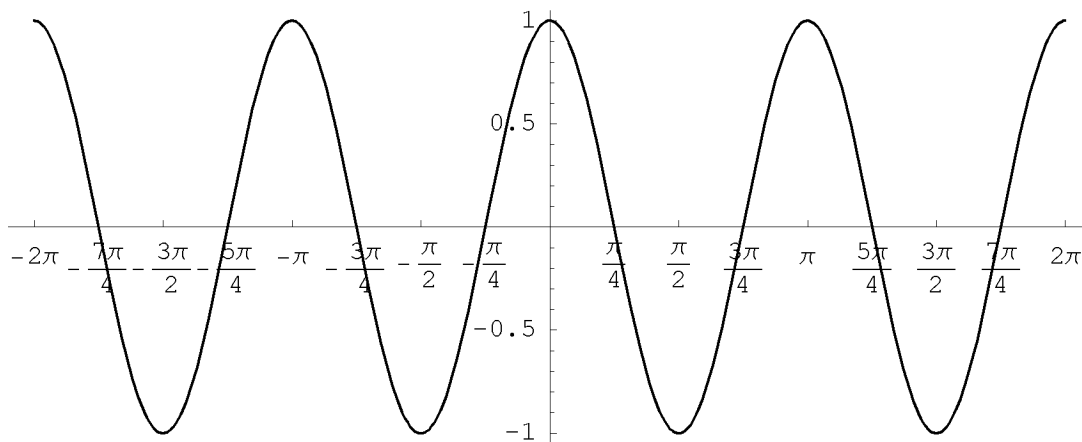
[1] The graph crosses the x-axis at $-\frac{5\pi}{3}$, $\frac{\pi}{3}$, $\frac{7\pi}{3}$. It has a maximum of 1.5 and a minimum of -1.5 .



The function of graph [1] is:

Answer $y = \frac{3}{2} \sin \frac{1}{2} \left(x - \frac{\pi}{3} \right)$

[2] The graph crosses the x-axis at $-\frac{5\pi}{4}, \frac{3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$. It has a maximum of 1.0 and a minimum of -1.0 .

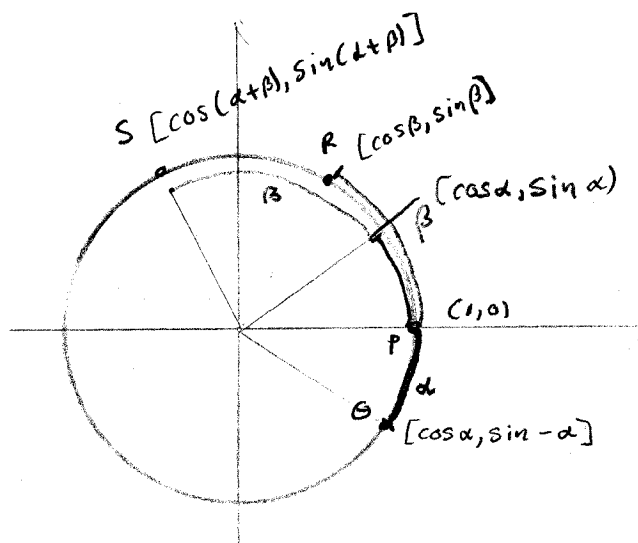


The function of graph [2] is:

Answer $y = \sin 2(x + \frac{\pi}{4})$

■ D. Proofs

[1] Prove any one of the addition theorems that involves only sines and cosines.



$$\widehat{PS} = \widehat{QR} \Rightarrow \overline{PS} = \overline{QR} \Rightarrow \overline{PS}^2 = \overline{QR}^2.$$

$$\begin{aligned}\overline{PS}^2 &= (\cos(\alpha + \beta) - 1)^2 + (\sin(\alpha + \beta))^2 = \cos^2(\alpha + \beta) - 2\cos(\alpha + \beta) + 1 + \sin^2(\alpha + \beta) \\ &= 2 - 2\cos(\alpha + \beta)\end{aligned}$$

$$\begin{aligned}\overline{QR}^2 &= (\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin(-\alpha))^2 \\ &= (\cos\beta - \cos\alpha)^2 + (\sin\beta + \sin\alpha)^2 \\ &= \cos^2\beta - 2\cos\alpha\cos\beta + \cos^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \sin^2\alpha \\ &= 2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta\end{aligned}$$

$$\overline{PS}^2 = \overline{QR}^2 \Rightarrow \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

□

[2] Prove: **$\sin 2\alpha = 2\sin\alpha\cos\alpha$**

$$\sin(2\alpha) = \sin(\alpha + \alpha) = \sin\alpha\cos\alpha + \sin\alpha\cos\alpha = 2\sin\alpha\cos\alpha \quad \square$$

[3] Prove: **$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$**

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha \quad (\text{EQ1})$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha \quad (\text{EQ2})$$

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin\alpha\cos\beta \quad (\text{EQ1}) + (\text{EQ2})$$

$$\therefore \sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

□

[4] Prove: **$\cos 2x = 2\cos^2 x - 1$**

$$\begin{aligned}\cos 2x &= \cos(x+x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x \\ &= \cos^2 x - (1 - \cos^2 x) = 2 \cos^2 x - 1 \quad \square\end{aligned}$$

■ E. Answer the following

[1] Find the maximum and minimum values of $y = \sin \theta + \sqrt{3} \cos \theta$ and state the values of θ , $0 \leq \theta < 2\pi$, at which they occur.

Let point P in the coordinate plane have coordinates $P(1, \sqrt{3})$. Then distance origin to P is $\overline{OP} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$. If \overline{OP} makes angle α with positive x-axis, then

$$\sin \alpha = \frac{\sqrt{3}}{2} \text{ and } \cos \alpha = \frac{1}{2}.$$

$$\sin \theta + \sqrt{3} \cos \theta = 2 \left(\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta \right) = 2(\cos \alpha \sin \theta + \sin \alpha \cos \theta) = 2 \sin(\alpha + \theta).$$

$$\text{Now, } \tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \implies \alpha = \tan^{-1} \sqrt{3} \implies \alpha = \frac{\pi}{3}.$$

$$\text{Thus, } 2 \sin(\alpha + \theta) = 2 \sin\left(\theta + \frac{\pi}{3}\right) = y.$$

$$\therefore y_{\max} = 2 \text{ when } \theta + \frac{\pi}{3} = \frac{\pi}{2} \implies \theta = \frac{\pi}{6}.$$

$$\therefore y_{\min} = -2 \text{ when } \theta + \frac{\pi}{3} = \frac{3\pi}{2} \implies \theta = \frac{7\pi}{6}.$$

[2] Solve for θ , $0 \leq \theta \leq 2\pi$, if $\sin \theta = 2 \sin^2 \theta$.

$$2 \sin^2 \theta - \sin \theta = 0 \implies \sin \theta (2 \sin \theta - 1) = 0 \implies \theta = \frac{\pi}{2} \text{ or}$$

$$\sin \theta = 0 \implies \theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}.$$

$$2 \sin \theta - 1 = 0 \implies \sin \theta = \frac{1}{2} \implies \theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}.$$

[3] Solve for x , if $5 \cos 3x - 1 = 4 \cos 3x + 1$

$$5 \cos 3x - 1 = 4 \cos 3x + 1 \Rightarrow \cos(3x) = 2 \Rightarrow \text{no solution.}$$

[4] Solve for x , if **$\cos 2x - 1 = 3 \cos 2x - 1$**

$$2 \cos[2x] = 0 \Rightarrow \cos[2x] = 0 \Rightarrow 2x = \frac{\pi}{2} \text{ or } 2x = \frac{3\pi}{2}$$

$$\text{Thus, } S = \left\{ x \mid x = \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z} \right\} \cup \left\{ x \mid x = \frac{3\pi}{2} + 2n\pi, n \in \mathbb{Z} \right\}$$

[5] Solve for θ , if **$\sin 4\theta = -\sin 3\theta$**

$$\sin 4\theta = -\sin 3\theta \Leftrightarrow \sin 4\theta = \sin -3\theta$$

$$\Leftrightarrow 4\theta = -3\theta + 2n\pi \text{ or } 4\theta = \pi + 3\theta + 2n\pi$$

$$\Leftrightarrow \theta = \frac{-2n\pi}{7} \text{ or } \theta = \pi + 2n\pi = (2n-1)\pi, \text{ where } n \in \mathbb{Z}$$

Formulae